A Wavelet Analysis of Output Fluctuations in the Japanese Economy*

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Abstract

Though the Japanese economy has hitherto repeated the business cycle in the postwar era, the phases change over time. This paper uses wavelet analysis to identify the causes of output growth volatility in terms of final uses of GDP. In nearly half a century, it is shown that time-varying characteristics for output growth volatility are not uniform across frequencies. We demonstrate that domestic demand and its constituents account for much of these phenomena. Especially at business cycle frequencies, discretionary fiscal policies are shown to mitigate output fluctuations to some extent. Only in the recent Great Recession, external demand has a significant impact on the aggregate economy at business cycle frequencies as well as at higher frequencies, to the extent even of entailing the largest decline in output in the postwar period .

Keywords: Wavelet, Output growth volatility, Final uses of GDP

JEL classification: E32

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1 Introduction

The business cycle results from a variety of causes, and the extent to which output growth fluctuates in short-run is time-dependent. There is a large body of research exploring how the business cycle occurs. As is well known, for example, a marked decline in the volatility of output growth is observed in many industrialized countries. In the U.S. case, as studied in McConnell and Perez-Quiros (2000) and many others, the volatility of real output growth has declined dramatically from around the mid-1980s to the recent financial crisis and subsequent Great Recession. This lower volatility is now commonly referred to as the Great Moderation, and many authors provided numerous explanations for its cause, famous examples of which include good luck, good policy, and good practices.[1)]

In the Japanese case, Kimura and Shiotani (2009) used an approximate bandpass filter developed by Baxter and King (1999) to decompose the month-to-month variance in output growth by frequency and characterized the changing volatility at different frequencies. Their results are consistent with good practices hypothesis as the explanation for the decline in output fluctuations in the 1980s. In particular, they reported such a structural break at business cycle frequencies. More recently, Ko and Murase (2013) employed a time-varying vector autoregression model and showed that technology shocks yield the output volatility.

In this paper, we trace the source of the changing volatility of quarter-to-quarter fluctuations in Japanese output growth to the principal components of demand side (i.e., final uses of GDP). In relation to the above good policy hypothesis, an empirical evaluation of fiscal policy is conducted.[2)] While our empirical strategy is quite straightforward in the sense that we make a simple comparison between the volatility of GDP growth and that of the demand components, the frequency-domain perspective is introduced in order to distinguish fluctuations at business cycle frequencies as in Kimura and Shiotani (2009). We further assume that the volatility evolves over time at each periodic component of economic variables. To do this, we utilize a wavelet tool, the wavelet power spectrum, thereby identifying both changes over time and differences across frequencies in economic variables within a unified framework. As Aguiar-Conraria and Soares (2013) apply in the U.S. case, the wavelet power spectrum is particularly well suited to study business fluctuations and can be easily adapted to consider the time-

1) See, e.g., Stock and Watson (2002) and Ahmed et al. (2004) for good luck hypothesis, Clarida et al. (2000) and Primiceri (2005) for good monetary policies, and McConnell and Perez-Quiros (2000) for good practices.

2) See, e.g., Miyao (2000) for an empirical analysis of the Japanese traditional monetary policy, and Honda (2014) for the nontraditional monetary policy.

varying features at each periodic component.[3)]

We find that time-varying characteristics for output growth volatility are not uniform across frequencies in nearly half a century. For example, the results reveal that the output growth is moderated twice in nearly half a century, but such phenomena occurred at different frequencies whose range is narrower than that of typical business cycle frequencies. While it is difficult to conclusively pinpoint the causes of business fluctuations, we demonstrate that domestic demand and its constituents account for much of the variability of output growth. Specifically, whereas private consumption is shown to be relatively stable and not to be the chief cause of output fluctuations, private investment is shown to be relevant to output fluctuations. Especially at business cycle frequencies, discretionary fiscal policies are shown to stabilize output fluctuations to some extent. Only in the recent Great Recession, external demand has a significant impact on the aggregate economy at business cycle frequencies as well as at higher frequencies, to the extent even of entailing the largest decline in output in the postwar period.

The outline of the paper is as follows. In Section 2 we summarize the wavelet method used in the analysis. After providing a detailed look at the changing volatility of output growth, in Section 3 we analyze the causes of output fluctuations in terms of final uses of GDP and present the results. Section 4 offers a brief conclusion and suggestions for further work.

2 Wavelet Method

In this section we outline the wavelet approach that we use to analyze the output volatility in time-frequency space. When explaining the features of the wavelet analysis, it is useful to begin by seeing the essence of the Fourier analysis that is widely utilized in conventional works in economics. Even in the business cycle literature, the Fourier spectral analysis is widely applied and can be found in Granger (1966), King and Watson (1996), and others.

Given a time series $x(t)$, the Fourier transform is expressed as follows:

$$F_x(\omega) \;=\; \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt,$$

where $i=\sqrt{-1}$ is the imaginary unit, ω is the angular frequency that is related to the common frequency f, such that $f=\frac{\omega}{2\pi}$, and the units for ω are radians per unit time. Noting Euler's

3) To the best of my knowledge, in the business cycle literature, the wavelet analysis was initially applied by Raihan et al. (2005). Subsequently, multiple applications are provided by, e.g., Yogo (2008), Aguiar-Conraria and Soares (2011) and Rua (2013).

formula

$$e^{i\omega t} \;=\; \cos(\omega t) + i\sin(\omega t),$$

one can understand that each periodic component of the time series is assumed to be formulated as trigonometric function in the Fourier transform. In the Fourier framework, therefore, we are not able to capture changes over time in each frequency.

Incidentally, an ideal band-pass filter, which isolates only frequencies in the ranges $\omega_h < |\omega| < \omega_l$, is implemented as follows. Supposing the conventional interval of the angular frequency, $-\pi < \omega < \pi$, the extracted periodic components $x^B(t)$ are found by the inverse Fourier transform

$$x^B(t) \;=\; \frac{1}{2\pi}\int_{-\pi}^{\pi} B(\omega)F_x(\omega)e^{i\omega t}d\omega,$$

where

$$\begin{aligned} B(\omega) &= 1 \quad \text{for} \;\; \omega \in [\omega_l, \omega_h] \cup [-\omega_h, -\omega_l], \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

Likewise, the basic procedure of the other filters, such as low-pass and high-pass filters, are carried out in a similar manner by setting $B(\omega)$ depending on the interest of frequencies.

The amplitude of the Fourier transform yields the power spectrum

$$PS_x(\omega) \;=\; |F_x(\omega)|^2,$$

which depends only on the angular frequency, due to the above feature of the Fourier transform. The power spectrum provides useful information about how much of each periodic component exists in the time series, but on the other hand, it contains no information about how each periodic component changes over time. Contrastingly, as can be seen below, the wavelet transform enables a more flexible approach.

We now turn to the wavelet framework. In line with recent studies that apply the wavelet analysis to economics, such as Aguiar-Conraria et al. (2012), Caraiani (2012), and Rua (2012), we consider the continuous wavelet transform. The continuous wavelet transform of the time series $x(t)$ for a mother wavelet ψ is given by:

$$W_x(\tau, s) \;=\; \int_{-\infty}^{\infty} x(t)\tilde{\psi}^*\left(\frac{t-\tau}{s}\right)dt, \quad s, \tau \in \boldsymbol{R}, s \neq 0,$$

where

$$\tilde{\psi}_{\tau,s}(t) = \frac{1}{\sqrt{|s|}}\psi\left(\frac{t-\tau}{s}\right),$$

s is the scaling factor that determines the length of the wavelet, τ is the translation parameter that represents its location, and asterisk denotes complex conjugation.

For the purpose of performing the analysis, the mother wavelet must be specified. If we consider the most commonly used mother wavelet of the complex-valued wavelets, the called Morlet wavelet

$$\begin{aligned}\psi_{\omega_0}(t) &= \pi^{-1/4}\left(e^{i\omega_0 t} - e^{-\omega_0^2/2}\right)e^{-t^2/2} \\ &\simeq \pi^{-1/4}e^{i\omega_0 t}e^{-t^2/2} \quad \text{for} \quad \omega_0 \geq 5,\end{aligned}$$

then it is easy to see the relationship between frequencies f and the scaling factor s such that $f \simeq 1/s$ under the condition $\omega_0 = 6 \simeq 2\pi$. Another advantage of choosing the Morlet wavelet is that it has optimal joint time-frequency concentration.[4)] For these reasons, we assume the Morlet wavelet and $\omega_0 = 6$ as in Aguiar-Conraria et al. (2012).

Unlike the Fourier analysis, the wavelet methodology is quite flexible since the wave is within a Gaussian envelope. When we are interested in high frequency features, the wavelet can be localized by making the scaling factor smaller, and vice versa. Further, we are able to capture time-varying behavior of the time series by changing the value of the translation parameter.

In analogy with the Fourier case, the amplitude of the wavelet transform provides a useful tool for measuring the contribution to the variance of the series $x(t)$ around each time and frequency. That is, one can define the wavelet power spectrum as

$$WPS_x(\tau,s) = |W_x(\tau,s)|^2.$$

In contrast to the Fourier power spectrum, $WPS_x(\tau,s)$ indicates how the strength of the time series $x(t)$ is distributed not only across frequencies but also over time.

3 Empirical Results

The data used are quarterly observations in the period from 1955:3 to 2010:1. We obtained all data from the website of the Cabinet Office for the Government of Japan. All the series are sea-

4) The other properties of the Morlet wavelet are mentioned in Aguiar-Conraria and Soares (2013).

sonally adjusted and constructed by linking the System of National Accounts 1968（68SNA）with the System of National Accounts 1993（93SNA）. We used quarterly growth rate in real terms. In all our numerical computations for the above-mentioned wavelet measures, we used ASToolbox provided by Luis Aguiar-Conraria and Maria Joana Soares.

At this point, before detailing our wavelet results, we define the range of business cycle frequencies. Though, among others, the well-cited papers by Baxter and King（1999）and Christiano and Fitzgerald（2003）consider business cycles as periodic components whose frequencies lie in the range from 1.5 to 8 years per cycle, the period of oscillation is based on the U.S. business cycles. Since our interest here is in the Japanese economy, we adopt the definition of business cycles on the basis of the "date of business cycles" by the Cabinet Office for the Government of Japan. To the extent that our sample period is concerned, according to this definition, the range of the Japanese business cycles is approximately between 2.5 and 7 years. Hence, we consider the business cycle frequency band as 2.5-7 years.

3.1 Identifying Output Fluctuations across Time and Frequencies

Prior to disaggregated analyses for final uses of GDP, we characterize changes in the process for Japanese output growth in the postwar era. Our wavelet approach is demonstrated in Figure 1. The upper panel plots real GDP growth, and the lower panel plots the wavelet power spectrum in the time-frequency space. Note that the vertical axis in the lower panel corresponds to the frequency, but it is converted to time units（years）. To assess the statistical significance of the wavelet power spectrum, Monte Carlo simulations are performed, such that the surrogate series are constructed by a fitted ARMA(1,1) model with errors from a Gaussian distribution.

Disregarding all frequencies outside business cycle frequencies, from a simple visual inspection of Figure 1, we first notice that there are two highly-volatile periods, both of which are statistically significant at the 0.05 level. The first volatile period is located in the early 1970s and corresponds to the period including the first oil crisis in 1973. It should be noted that the abrupt economic slowdown precipitated by the first oil crisis in 1973 is reflected in the significant region where the cycle is between a half and 1 year. These increased volatility is consistent with the results of Gallegati and Gallegati（2007）, who find that volatility in the 1970s increases in the G-7 industrialized countries. They moreover show that such increases in the 1970s are caused not by country-specific shocks but by common disturbances hitting the international economy, probably due to the oil shocks.

While it is under the effect of the cone of influence for the most part, the second volatile period is the recent Great Recession in the late 2000s. From the upper panel, the peak-to-trough fall in output growth is shown to be one of the largest in the postwar era. It is noteworthy that

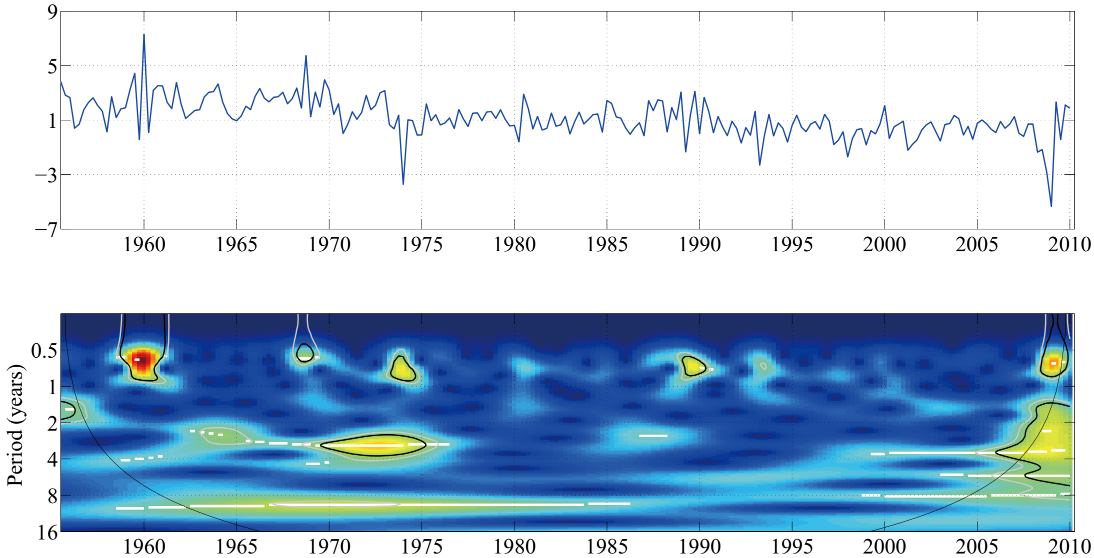


Figure 1: GDP

Notes: The lower panel plots the wavelet power spectrum, which ranges from blue (low power) to red (high power). The thick and thin black contours respectively stand for the 5% and 10% significance levels. The white lines represent the maxima of the wavelet power spectrum and the black line represents the cone of influence.

there is an important difference between these two significant regions: that is, the significant region of the first highly-volatile period is larger than that of the second period. More specifically, whereas the significant power ranges only at 3-4 years in the former case, it does at all business cycle frequencies in the latter case. This aspect can be a key factor when identifying the causes of the increased volatility.

Conversely, from the low power regions of Figure 1, one can see that the real output growth is moderated at business cycle frequencies from the late 1970s to the mid-2000s in the postwar era. If higher frequencies as well as business cycle frequencies are taken into account, as Ko and Murase (2013) state, we detect the timing of the Great Moderation in the Japanese economy in two subsample periods: from the late 1970s to the early 1980s and from the 1990s to the mid-2000s. In other words, except for the late 1980s, there is no significant region at the 0.05 level in the period from the late 1970s through the mid-2000s. It is certainly true that the economy is markedly less volatile in these subsamples than in prior periods, but we find that these two moderation phenomena differ by frequency. The first moderation occurs at all business cycle frequencies, but on the other hand, the wavelet power spectrum shows a few spikes (albeit without any significant region) at business cycle frequencies in the second moderation.

3.2 Sources of Output Fluctuations

In our next step, we search for the causes of the changing output volatility in terms of disaggregated demand components of GDP. The preceding subsection identifies volatile and less volatile periods in output fluctuations. In order for this identification to serve as useful information to investigate the causes of the variability of output growth, we must find similar regions of wavelet power spectrum in time-frequency plane for each component of GDP.

The final uses of GDP are divided into two broad categories: domestic demand and external demand (net exports). We start with a comparison between GDP and domestic demand. As before domestic demand and the wavelet power spectrum are shown in Figure 2. With the exception of the recent worldwide recession, the wavelet power spectrum in the lower panel resembles that of Figure 1, indicating that the output growth is influenced profoundly by domestic demand rather than external demand.[5] Because of this similarity, in what follows, we focus on the volatility in the constituents of domestic demand.

Domestic demand is the total of two variables: private demand and public demand. Concerning private demand and its constituents (i.e., private consumption, private residential investment, and private non-residential investment), their wavelet power spectra are reported in Figure 3. The wavelet power spectrum of private demand in Panel A of Figure 3 resembles that of

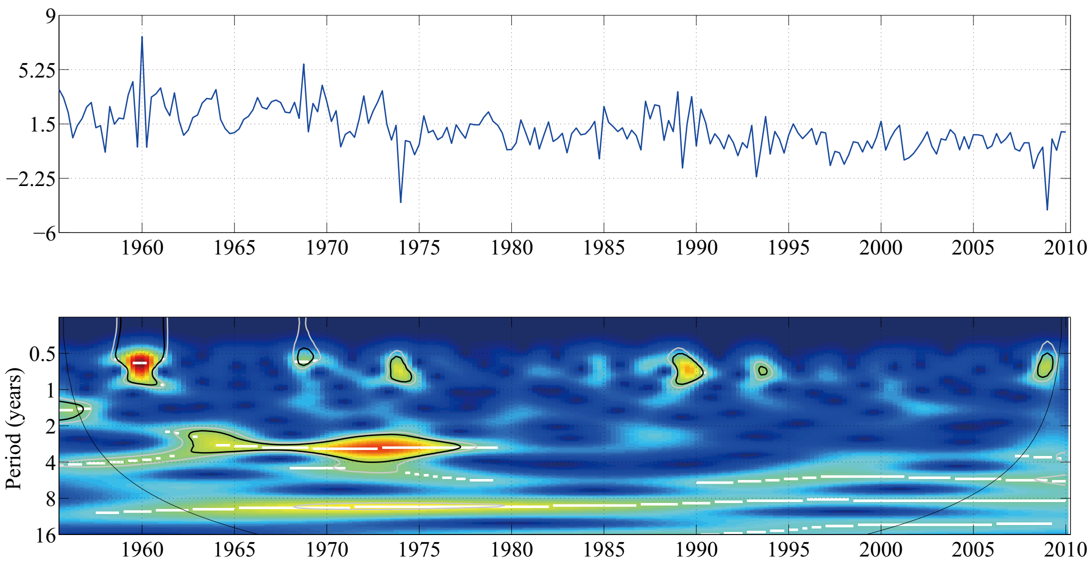


Figure 2: Domestic Demand

Notes: Same as Figure 1

5) In other words, as expected, the recent slowdown in Japan is triggered by an extreme decline in external demand. See Appendix for the results of external demand.

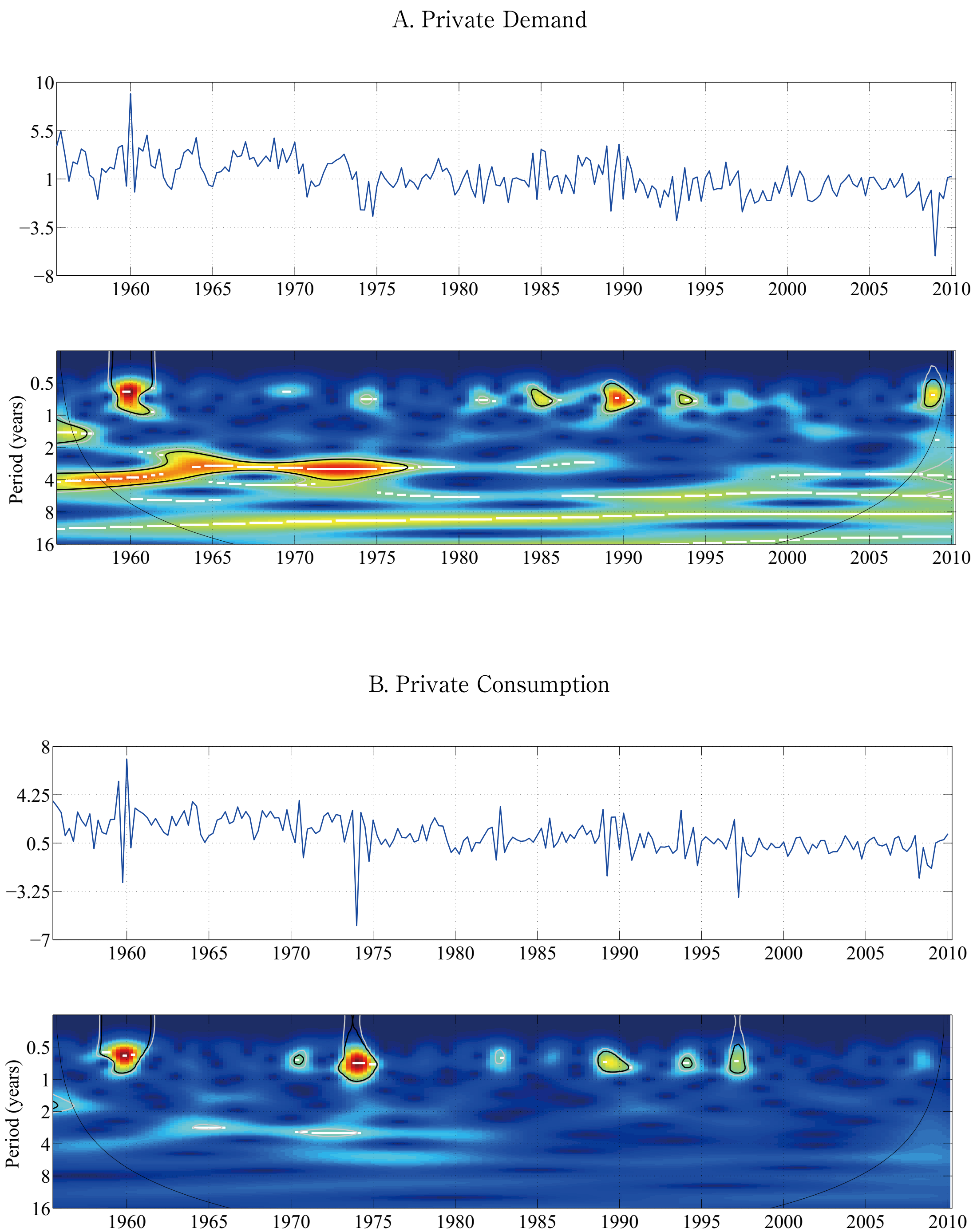
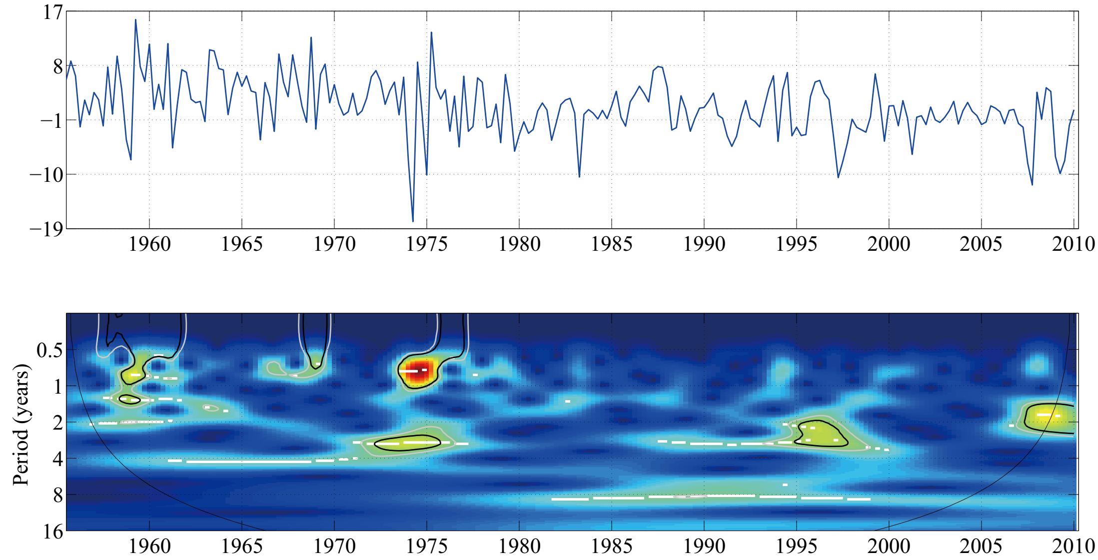


Figure 3: Private Demand and Its Constituents

Notes: Same as Figure 1

C. Private Residental Investment



D. Private Non-Residential Investment

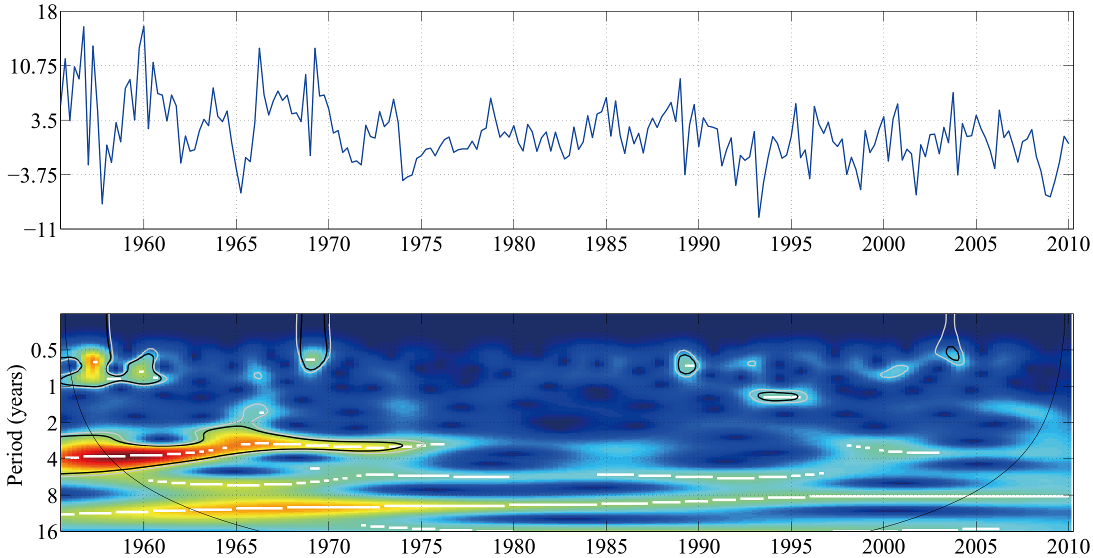


Figure 3: Private Demand and Its Constituents（Continued）

domestic demand as well as GDP to some degree. In particular, the large decline in output volatility in the mid-1970s appears to be mirrored in private demand.

The results for the constituents of private demand differ from one another. First, from Panel B of Figure 3, we can hardly see significant power in the changing variability of private consumption at business cycle frequencies. Unlike private demand and domestic demand, at business cycle frequencies, there is no significant region at the 0.05 level in the mid-1970s in private consumption.[6] Such an unvaried wavelet power spectrum suggests a well-known fact that private consumption that constitutes more than half of GDP is relatively stable over business cycle fluctuations. Second, from Panel C of Figure 3, we find a significant region in private residential investment in the vicinity of the mid-1970s corresponding to high wavelet power of private demand, domestic demand, and GDP. Of these, the volatile region in time-frequency space overlaps that of GDP. Thus, private residential investment appears to account for the first volatile period of output fluctuations. Third, from Panel D of Figure 3, we detect a relatively large significant region at business cycle frequencies from the 1960s to the mid-1970s, implying that private non-residential investment is moderated in the vicinity of the mid-1970s. This region overlaps that of private demand, and the decline corresponds to the first moderation. Hence, the reduced volatility of private non-residential investment after the mid-1970s appears to bring about the increased aggregate stability from the late 1970s to the mid-1980s.

Up to this point we have explored the behavior of the private sector. We next turn to the results of public demand that are related to fiscal policies. As already described earlier, domestic demand is divided into private demand and public demand, and consequently public demand is only remaining component of domestic demand. The discrepancy between domestic demand and private demand, therefore, can be attributed to public demand and its constituents. Thus, unless government's behavior affects the economy in an obvious fashion, the discrepancy is not observed. If government spendings move countercyclically according to Keynesian demand management principles, as is likely, then we would expect that the wavelet power spectrum for domestic demand is lower than the power for private demand at business cycle frequencies.[7]

Turning back to a comparison between Figure 2 and Panel A of Figure 3, we see that the wavelet power spectrum for private demand exhibits a large significant region at business cycle frequencies until the mid-1970s, but that of domestic demand is lower before the late 1960s. From this, taking into account the view that the frequency band of countercyclical policy is

6) The peak-to-trough fall around 1973 is over a quite short period in private consumption, so that the wavelet power spectrum is statistically significant at high frequencies.

7) Ahmed et al. (2004) express the same motivation and utilize a frequency-domain technique to analyze the Great Moderation in the United States.

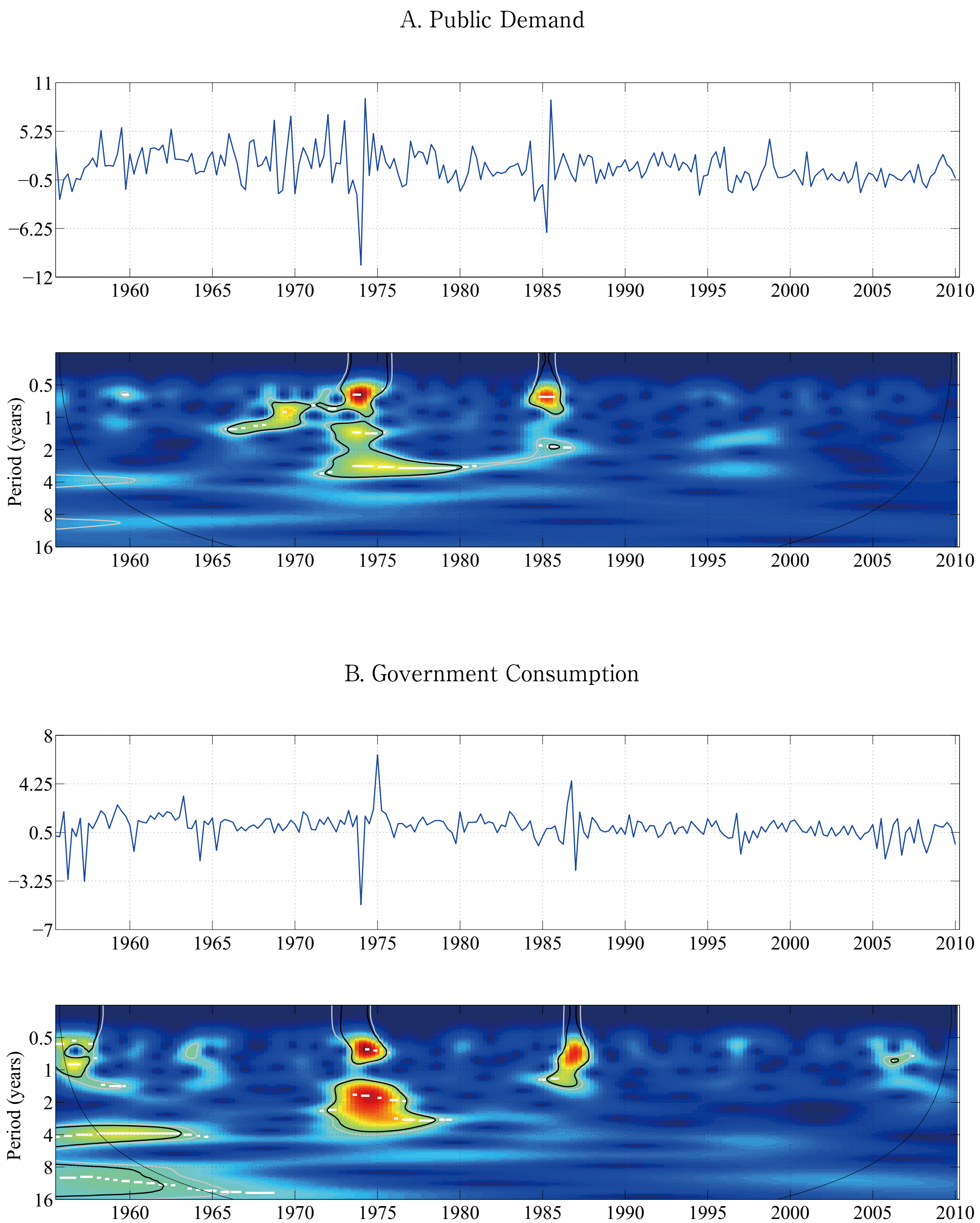


Figure 4: Public Demand and Its Constituents

Notes: Same as Figure 1

C. Public Investment

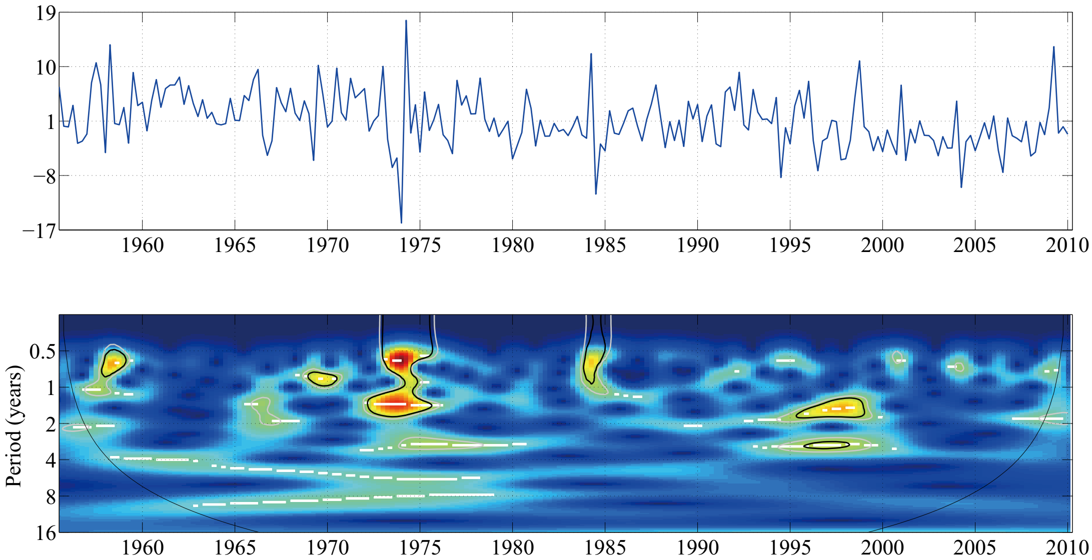


Figure 4: Public Demand and Its Constituents （Continued)

conterminous with that of business cycles, one can indicate the possibility that government tames the business cycle in this period. It is also possible that the same logic applies to the other period. For example, while there are two spikes in the 1980s in the case of private demand, most of them disappear in domestic demand.

In order to show more formally how government spendings have an impact on the economy, Figure 4 plots public demand and its constituents with the wavelet power spectra. For the most part these results reinforce the above suggestions. Public demand is divided into government consumption and public investment. One common feature to these three variables is that there is a high power region in the vicinity of the first oil crisis. On the other hand, one can observe massive differences between government consumption and public investment. Looking at Panel B of Figure 4, government consumption, which is the second largest final use after private consumption, is generally less volatile and the wavelet power spectrum does not exhibit many regions of high power. In Panel C of Figure 4, public investment has a lot of ups and downs, and the wavelet power spectrum shows multiple spikes, especially at business cycle frequencies. The behavior of public investment over the business cycle appears to reflect a countercyclical pattern for smoothing output fluctuations. In fact, it is widely recognized that Japan's discretionary fiscal stimulus is primarily implemented by public investment.

The stabilization effects ensuing from public investment are compatible with the outcomes provided by Bayoumi（2001）, Ihori et al.（2003）, and others, who point out that the effect has declined since the 1990s. Reminding the above comparison between Figure 2 and Panel A of Figure 3, at business cycle frequencies, the wavelet power spectrum of domestic demand relatively weakens before the 1990s, but on the other hand, it differs little from that of private demand after the 1990s. However, it should be noted that, even after the 1990s, some subtle differences can be observed between these figures, implying that fiscal policies are not utterly ineffective. In summary, discretionary fiscal policy, notably public investment, has played some role in offsetting output fluctuations over the whole postwar period.

4　Conclusion

In this paper, we examined the Japanese output fluctuations in terms of demand components of GDP and argued that domestic demand, particularly private investment, has played a key role in explaining the output volatility, except for the recent Great Recession. Not surprisingly, only the recent dramatic decline in output is shown to be accompanied by a similar reduction in exports. We moreover argued that public investment rather than government consumption is implemented according to Keynesian demand management principles, so that output fluctuations are partly mitigated, especially before the 1990s. It is highly likely that most of the above outcomes cannot be uncovered by using the conventional time series analysis, including the frequency-domain analysis or some band-pass filters.

There are many related issues which can also be addressed by utilizing some wavelet techniques. First, when explaining the output volatility, an important role for monetary policy cannot be ruled out. While in this paper we have evaluated public expenditures, it is also necessary to examine the effects of monetary policy that influences the components of GDP. Thus, as in Aguiar-Conraria et al.（2008）, wavelet analysis of the relationship between monetary policy and macroeconomic variables should be pursued in the Japanese case. Second, the present analysis resorted to the wavelet power spectrum, but it is insufficient to complement the evidence of output fluctuations since information about the phase is lost by taking the absolute value. In other words, by means of the wavelet power spectrum, one cannot know whether the time series of interest rise or fall. For example, as a robustness check, we need to confirm whether or not public expenditures move in a countercyclical direction by undertaking a comparison of the phases between output and public expenditures. To this end, though we performed an univariate wavelet analysis, the multivariate framework is very useful as found in Aguiar-Conraria and Soares（2013）and others.

Appendix

This appendix presents the results of the wavelet power spectrum pertaining to external demand. In the System of National Accounts, it is conventional to perceive external demand as being equal to net exports (i.e., exports minus imports). Figure 5 displays the constituents of external demand, exports and imports, with the wavelet power spectra. From this, many differences between the two series can be clarified. Of these, an important point about output fluctuations is related to their peak-to-trough falls in the recent Great Recession. In exports, the recession entails a dramatic decline over a certain period. By comparison, in imports, it does a relatively modest decline over a shorter period than exports. The difference is also reflected in the wavelet power spectra, so that whereas there is a large significant region at business cycle frequencies as well as higher frequencies in exports, the power exhibits high values only at high frequencies in imports. It is also important to note that the high power region in exports overlaps that in output, implying that the recent Great Recession results mainly from a substantial reduction in exports.

A. Exports

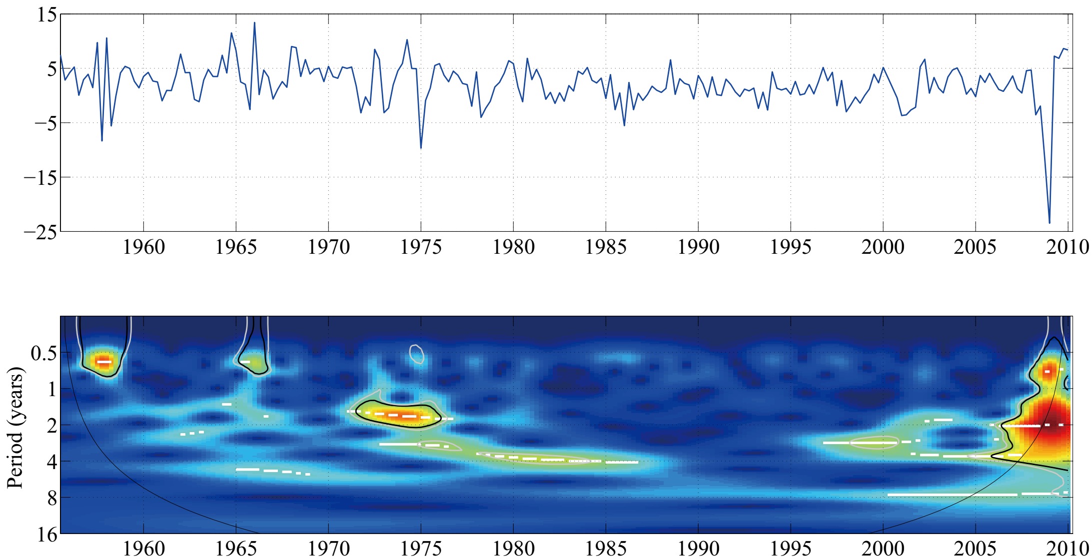


Figure 5: Exports and Imports

Notes: Same as Figure 1

B. Imports

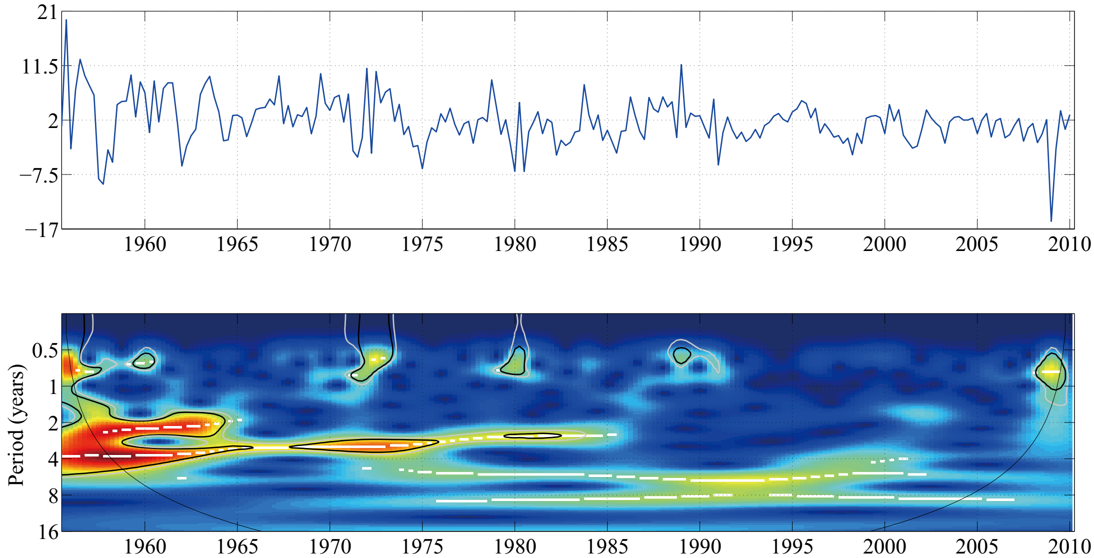


Figure 5: Exports and Imports（Continued）

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